Topic 5: Dimensional Analysis

for use after ***Comparing and Scaling*** **Investigation 3**

To convert a measurement from one unit to another, you can use a conversion factor. A **conversion factor** is a rate equal to 1. For example, 12 in. = 1 ft, so you can use the rate $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert feet to inches.

Problem 5.1

A. 1. Use the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert 100 feet to inches.

2. Use a conversion factor to convert 100 inches to feet.

B. 1. What conversion factor can you use to change seconds to minutes?

2. What conversion factor can you use to change minutes to seconds?

C. Which unit belongs in the denominator of the conversion factor, the given measurement or the resulting measurement?

Dimensional analysis is a method of checking the units that result from using conversion factors. You can use dimensional analysis to check whether your methods and answers are reasonable.

Problem 5.2

A. 1. Use conversion factors for hours to minutes and minutes to seconds to write a rate for converting hours to seconds.

2. Write a conversion factor for changing seconds to hours.

B. 1. You want to convert 1,000 seconds to hours. Which method below is correct?

$1{,}000 \text{ s} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 3{,}600{,}000 \text{ h}$

$1{,}000 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.28 \text{ h}$

2. You want to convert 240 miles per second to miles per hour. Which method below is correct?

$240 \frac{\text{mi}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 864{,}000 \text{ mi/h}$

$240 \frac{\text{mi}}{\text{s}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.67 \text{ mi/h}$

Exercises

1. The table shows equivalent measurements.

Measurements

Length in Meters	Length in Feet
1	3.28
2	6.56
3	9.84
4	13.12

a. Write a conversion factor for changing meters to feet.

b. Write a conversion factor for changing feet to meters.

c. How many feet equal 100 meters?

d. How would you find a conversion factor for changing square meters to square feet?

For Exercises 2–3 below, do parts (a) and (b).

a. Use a conversion factor to solve the problem.

b. Use dimensional analysis to check your answer.

2. Change 432 square inches to square feet.

3. Change 2,232 minutes to days.

4. You bike for 45 minutes at a rate of 10 mi/h. You turn around and return by the same route. Your return trip takes 30 minutes. What was your average speed over the entire trip?

5. *Density* is a unit rate. It is the mass of a substance per unit volume. The table gives data for the masses and volumes of four metal samples.

Valuable Metals

Metal	Mass (kilograms)	Volume (cubic centimeters)
Copper	8,930	1
Gold	9,660	0.5
Silver	20,980	2
Titanium	4,500	1

a. Which metal has the greatest density?

b. Convert the density of copper from 8,930 kilograms per cubic meter to grams per cubic centimeter. Use dimensional analysis to check that your answer is reasonable.

c. Which sample below shows the correct first step for converting the density of titanium to grams per cubic centimeter?

1. $4{,}500\ \frac{\text{kg}}{\text{m}^3} \times \frac{1{,}000\ \text{g}}{1\ \text{kg}} \times \frac{1\ \text{m}}{100\ \text{cm}} \times \frac{1\ \text{m}}{100\ \text{cm}} \times \frac{1\ \text{m}}{100\ \text{cm}}$

2. $4{,}500\ \frac{\text{kg}}{\text{m}^3} \times \frac{1\ \text{kg}}{1{,}000\ \text{g}} \times \frac{100\ \text{cm}}{1\ \text{m}} \times \frac{100\ \text{cm}}{1\ \text{m}} \times \frac{100\ \text{cm}}{1\ \text{m}}$

d. Write the density of titanium in grams per cubic centimeter.

Topic 5: Dimensional Analysis

At a Glance

PACING 1 day

Mathematical Goals

- Use conversion factors to convert units
- Use dimensional analysis to check units for reasonableness

Teaching Guide

Students may have trouble deciding whether to multiply or divide when converting between units. In Topic 5, students will learn to use dimensional analysis to check the units of a converted quantity. Students can avoid writing unreasonable answers by writing out all conversion factors and canceling units correctly.

After Problem 5.1, ask:

- *When you convert from a smaller unit to a larger unit, will the numerical result be greater than or less than the original measure?*
- *When you convert from a larger unit to a smaller unit, will the numerical result be greater than or less than the original measure?*
- *Is it easier to think of a length as 5 feet or 60 inches?*

Summarize Problem 5.2A by asking:

- *How do you know that the rate you found for converting hours to seconds is a conversion factor?*
- *How can you use the rate you found for converting hours to seconds to write a conversion factor for changing seconds to hours without writing the conversion factors for hours to minutes and minutes to seconds?*

After Problem 5.2B, ask:

- *How does dimensional analysis help you decide which conversion factors to use?*
- *How can you keep track of which units remain after you multiply by a conversion factor?*

Homework Check

When reviewing Exercise 1, ask:

- *How can you use the table to write conversion factors for changing meters to inches and inches to meters?*

After reviewing Exercises 2–4, ask:

- *Can you write a conversion factor to change square inches to feet? Why or why not?*
- *Can you write a conversion factor to change from minutes to feet? Why or why not?*

Vocabulary

- conversion factor
- dimensional analysis

Assignment Guide for Topic 5

Core 1–5

Answers to Topic 5

Problem 5.1

A. 1. 1,200 inches

2. $8\frac{1}{3}$ ft

B. 1. $\frac{1 \text{ min}}{60 \text{ s}}$

2. $\frac{60 \text{ s}}{1 \text{ min}}$

C. the given measurement

Problem 5.2

A. 1. $\frac{3{,}600 \text{ s}}{1 \text{ h}}$

2. $\frac{1 \text{ h}}{3{,}600 \text{ s}}$

B. 1. the second method, 0.28 h

2. the first method, 864,000 mi/h

Exercises

1. a. $\frac{3.28 \text{ ft}}{1 \text{ m}}$

b. $\frac{1 \text{ m}}{3.28 \text{ ft}}$

c. 328 ft

d. Square the conversion factor for changing meters to feet.

2. a. 3 ft^2

b. Check students' work. Sample:
$432 \text{ in.}^2 \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ ft}}{12 \text{ in.}}$

3. a. 1.55 days

b. Check students' work. Sample:
$2{,}232 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ h}}$

4. 12 mi/h

5. a. gold

b. 8.93 g/cm^3

c. 1

d. 4.5 g/cm^3